

Supersymmetry breaking, R-symmetry breaking and metastable vacua

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ABSTRACT: Models of spontaneous supersymmetry breaking generically have an R-symmetry, which is problematic for obtaining gaugino masses and avoiding light R-axions. The situation is improved in models of metastable supersymmetry breaking, which generically have only an approximate R-symmetry. Based on this we argue, with mild assumptions, that metastable supersymmetry breaking is inevitable. We also illustrate various general issues regarding spontaneous and explicit R-symmetry breaking, using simple toy models of supersymmetry breaking.

KEYWORDS: Supersymmetry Phenomenology, Supersymmetry Breaking.

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1. Introduction

Theories of spontaneous F-term supersymmetry breaking generically have a global $U(1)_R$ symmetry [1, 2]. The argument of [2] ties spontaneous supersymmetry breaking with the existence of an exact R-symmetry. This argument can easily be extended to tie the existence of an approximate R-symmetry with metastable supersymmetry breaking.

More explicitly, a theory with an approximate R-symmetry has a small parameter ϵ , such that for $\epsilon = 0$ the theory has an R-symmetry, but for nonzero ϵ this symmetry is broken. Following [2], for $\epsilon = 0$ the theory breaks supersymmetry. We assume that, as in all known examples, this happens in a compact space of vacua. Now we turn on a small but nonzero ϵ . Clearly, the small effects of nonzero ϵ cannot ruin the supersymmetry breaking ground states. All they can do is slightly deform the expectation values in these states. However, because the theory with nonzero ϵ does not have an R-symmetry, it follows from [2] that it must have supersymmetric ground states. For small nonzero ϵ , these supersymmetric ground states are at field expectation values of order an inverse power of ϵ . Therefore, we conclude that for nonzero but small ϵ supersymmetry is broken in a metastable state. The longevity of this state is guaranteed for small ϵ because then the supersymmetric vacua are very far in field space and the tunneling to them is suppressed. For a recent review of such models and for an extensive list of references, see [3].

Now, let us turn to very basic phenomenological constraints. In order to have nonzero Majorana gaugino masses, the R-symmetry should be broken. This breaking can be either explicit, or spontaneous, or both explicit and spontaneous.

One possibility is that the theory has an exact $U(1)_R$ symmetry which is spontaneously broken at the scale of supersymmetry breaking; this occurs, for example, in the (3, 2)

model [1]. This option leads to a massless Goldstone boson — an R-axion — in the spectrum, which is experimentally ruled out. This R-axion can acquire a mass if the R-symmetry is explicitly broken. Indeed, in any theory of gravity we expect that there are no global continuous symmetries; therefore high dimension operators, whose coefficients are $\mathcal{O}(1/M_p)$, will explicitly break the symmetry. A specific example of such an operator arises from the constant term in the superpotential which is needed in order to set the cosmological constant to zero (or a very small value) [4]. Depending on the details of the scales of supersymmetry and R-symmetry breaking, this contribution to the R-axion mass might or might not be sufficient to be compatible with the various experimental constraints.

Here, we will not consider scenarios where the $U(1)_R$ symmetry breaking is entirely due to gravity. If we decouple gravity, then we must include explicit R-breaking terms in the field theory. Then, the argument above shows that the supersymmetry breaking ground state must be metastable. We conclude that, with some mild assumptions, low energy supersymmetry breaking requires that we live in a metastable state! This general observation is consistent with the fact that all known realistic models of supersymmetry breaking which do not involve gravity (starting with the seminal work of [5]) lead to a metastable state. We see that this fact is not just an embarrassing nuisance, which perhaps can be avoided with more ingenious model building — instead, metastability is inevitable.

Accepting metastability, even in the supersymmetry breaking sector of the theory, is helpful because it makes it much easier to construct models, as seen e.g. in [6]. In fact, the recent work of [7] suggests that metastable supersymmetry breaking is generic in field theory and string theory. Variants, and other models with metastable vacua have been presented, e.g. in [8–14].

In this paper, we illustrate these and other issues using toy models of both explicit and spontaneous R-symmetry breaking. Our examples are all based on variants of the O’Raifeartaigh model, which is the simplest example of renormalizable spontaneous F-term supersymmetry breaking. Recall that the original model [15] has three chiral superfields, X , ϕ_1 , and ϕ_2 , with canonical Kähler potential $K = K_{\text{can}} = X\bar{X} + \phi_1\bar{\phi}_1 + \phi_2\bar{\phi}_2$, and superpotential

$$W_{O'R} = \frac{1}{2}hX\phi_1^2 + m\phi_1\phi_2 + fX. \tag{1.1}$$

This theory has a $U(1)_R$ symmetry, with $R(X) = R(\phi_2) = 2$ and $R(\phi_1) = 0$. It has a classical pseudo-moduli space of supersymmetry breaking vacua, with arbitrary $\langle X \rangle$. At one-loop, this degeneracy is lifted [16], and the vacuum is at the origin of the pseudo-moduli space, with supersymmetry broken, and the $U(1)_R$ symmetry unbroken. On general grounds [2], adding a *generic*, R-symmetry breaking operator

$$\delta W = \epsilon f(\phi_1, \phi_2, X) \tag{1.2}$$

to the superpotential (1.1) will restore supersymmetry. However, if the coefficient ϵ of that operator is small, we expect the local analysis of the supersymmetry-breaking vacuum to be unaffected. Moreover, we expect the lifetime of the now metastable vacuum to be parametrically large in the small ϵ limit. In our first example, we illustrate this general

connection between approximate R-symmetry and metastable supersymmetry breaking by taking $\delta W = \frac{1}{2}\epsilon\phi_2^2$ and analyzing the resulting vacuum structure in detail.¹

For our second example, we will analyze a class of toy models for spontaneous R-symmetry breaking, also based on O’Raifeartaigh models. Because the pseudo-moduli X of the tree-level theory have $R(X) = 2$, the R-symmetry will be spontaneously broken if the quantum effective potential for the pseudo-moduli has a minimum away from the origin, $\langle X \rangle \neq 0$. To achieve this, we generalize (1.1) to include a symmetry G . For simplicity, we will focus on $G = \text{SO}(N)$, with the fields ϕ_i in the fundamental representation. (The qualitative features of our analysis should carry over easily to other groups and representations.) Weakly gauging G can lead to a vacuum with $\langle X \rangle \neq 0$, provided that X couples to the gauge group, and the gauge group is spontaneously broken.²

The fact that a generalized O’Raifeartaigh model, augmented with a gauge symmetry G , can lead to $\langle X \rangle \neq 0$ has been known for decades [18]. In the original example of [18], the idea was to use gauge interactions to generate an “inverted hierarchy” with $\langle X \rangle$ exponentially larger than the scales in the tree-level potential. In this limit, the leading contribution to the effective potential for the pseudo-modulus X is related to its anomalous dimension:

$$V_{\text{eff}}(X) \approx \gamma \log \left(\frac{|X|^2}{M_{\text{cutoff}}^2} \right) V_0 \quad \text{for } X \text{ large,} \quad (1.3)$$

where V_0 is the tree-level vacuum energy along the pseudo-moduli space. When several fields have non-zero F-terms on the pseudo-moduli space, γ is the anomalous dimension of a particular linear combination of them, as we will illustrate later in examples. In general, γ is a combination of the gauge and Yukawa couplings, given at one loop by

$$\gamma^{(1)} = c_h h^2 - c_g g^2 \quad (1.4)$$

with c_h and c_g positive numbers. If the pseudo-modulus is charged under the gauge group, then $c_g \neq 0$ and there is a negative contribution to the one-loop anomalous dimension coming from the gauge interactions. If the ratio g/h is sufficiently large, γ is negative, and the potential slopes down at large X . This is the situation considered in [18]. Whether X is stabilized at a finite value, as opposed to having a runaway $X \rightarrow \infty$, hinges on the higher loop contributions to (1.3). This was analyzed e.g. in [19–21], where it was found that g/h needs to be sufficiently large to have $\gamma < 0$, but not too large in order to avoid runaway. In this phase, the spontaneous R-symmetry breaking occurs at a scale $\langle X \rangle$ which is exponentially larger than the supersymmetry breaking scale.

Motivated by an interest in low-scale gauge mediation, we will focus the analysis of our models on a different phase of the theory, where γ is positive, yet $\langle X \rangle$ is stabilized at small, but nonzero values. The existence of such a “non-hierarchical” phase depends on the details of the full Coleman-Weinberg effective potential,

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{STr } \mathcal{M}^4 \log \frac{\mathcal{M}^2}{M_{\text{cutoff}}^2}. \quad (1.5)$$

¹A similar toy model was considered in [8].

²It is also possible to spontaneously break R-symmetry in O’Raifeartaigh models without gauge interactions [17]. This interesting scenario will not be considered here.

(where \mathcal{M} are the classical, pseudo-moduli-dependent masses), and thus it is not at all guaranteed. As a result, this non-hierarchical phase has not been much discussed in the literature. (It was considered long ago in an early model building attempt [22], and it was also discussed recently in [9, 14].) Here we will present a broad class of examples of this phenomenon, and attempt the first systematic analysis of it.

More specifically, we will show that in our gauged $\text{SO}(N)$ models, the potential can have a minimum at small or intermediate $\langle X \rangle \neq 0$, but only if the ratio g/h takes values in a *small window* which is typically of size $\lesssim 0.1$. On general grounds, such a window, if it exists at all, cannot be too large, since for sufficiently large g/h the theory is in the inverted hierarchy phase (or has runaway), while for small g/h the Yukawa interactions dominate and the $\text{U}(1)_R$ remains unbroken.

In addition, we will generalize our models to include the case where only an $\text{SO}(n)$ subgroup of $\text{SO}(N)$ is gauged. Now the phase structure is more intricate, and depending on the couplings the symmetry group $\text{SO}(n) \times \text{SO}(N-n)$ can either be broken to $\text{SO}(n-1) \times \text{SO}(N-n)$ or $\text{SO}(n) \times \text{SO}(N-n-1)$ along the pseudo-moduli space.³ In the former case, the potential depends on g and there can be a minimum with $\langle X \rangle \neq 0$, while in the latter case the potential is independent of g and the minimum is at $\langle X \rangle = 0$. The reason is because, as mentioned above, the gauge group must be spontaneously broken along the pseudo-moduli space in order for the potential (1.5) to lead to $\langle X \rangle \neq 0$. This can be seen from the standard expressions for the classical mass matrices \mathcal{M} , which enter in (1.5). Because we consider situations without D -term supersymmetry breaking, the masses \mathcal{M} only depend on the gauge coupling g if the gauge group is Higgsed. If the gauge group is not Higgsed, then the potential (1.5) coincides with that for $g = 0$, and then the minimum of the potential is at $\langle X \rangle = 0$.

The outline of the paper is as follows. In section 2, we collect some general facts and formulas about O’Raifeartaigh-type models. We focus on two different sub-classes of O’Raifeartaigh-type models. The first consists of straightforward generalizations of (1.1), while the second class consists of models with only cubic and linear superpotential interactions. The models of [18] and [7] belong in this second class. Section 2 will also set up the formalism that we will need for the examples studied in later sections. In section 3, we analyze the effects of adding a small R-symmetry breaking operator to the original O’Raifeartaigh model (1.1). Finally, section 4 contains our analysis of spontaneous R-symmetry breaking in the gauged $\text{SO}(N)$ and $\text{SO}(n) \subset \text{SO}(N)$ O’Raifeartaigh models.

2. General Remarks on O’Raifeartaigh models

In this section, we review some aspects of O’Raifeartaigh-type models. A large class of such models has r fields X_i , and s fields ϕ_j , with $r > s$, with a $\text{U}(1)_R$ symmetry under

³The question of dynamical vacuum alignment [23, 24] does not arise in this setup, because there are relevant interactions, which have no reason to respect the $\text{SO}(N)$ symmetry. In particular, the couplings in the superpotential will only respect the $\text{SO}(n) \times \text{SO}(N-n)$ symmetry. The symmetry breaking pattern in the vacuum is then determined by the tree-level values of the superpotential couplings, rather than dynamically.

which $R(X_i) = 2$ and $R(\phi_j) = 0$. We take these fields to have canonical Kähler potential, and superpotential

$$W = \sum_{i=1}^r X_i g_i(\phi_j). \tag{2.1}$$

For generic functions g_i , it is impossible for all $-F_{X_i}^\dagger = g_i(\phi_j)$ to vanish, so supersymmetry is generically broken.

The tree-level potential for the scalars is

$$V_{\text{tree}} = \sum_{i=1}^r |g_i(\phi_j)|^2 + \sum_{j=1}^s \left| \sum_{i=1}^r X_i \frac{\partial}{\partial \phi_j} g_i(\phi_j) \right|^2. \tag{2.2}$$

There is a pseudo-moduli space of vacua, given by the r fields X_i , subject to the s conditions

$$0 = \sum_{i=1}^r X_i \frac{\partial}{\partial \phi_j} g_i(\phi_j) \quad \text{for all } j = 1 \dots s. \tag{2.3}$$

For generic functions $g_i(\phi_j)$, this pseudo-moduli space has complex dimension equal to $r - s$. The X_i equations of motion are automatically satisfied on this pseudo-moduli space. The ϕ_j are determined by their equations of motion,

$$0 = \sum_{i=1}^r \overline{g_i(\phi_j)} \frac{\partial}{\partial \phi_j} g_i(\phi_j) \quad \text{for all } j = 1 \dots s, \tag{2.4}$$

which is generally satisfied for a discrete set of values $\phi_j = \phi_j^{(n)}$, some of which are local minima of the potential (others are saddle points). Whether or not a given solution $\phi_j = \phi_j^{(n)}$ is a local minimum can vary with the parameters in the $g_i(\phi_j)$.

To summarize, the tree-level potential generally has a pseudo-moduli space of supersymmetry breaking vacua, labeled by the $r - s$ complex dimensional space of expectation values of the X_i , subject to (2.3), along with some discrete choices $\phi_j^{(n)}$ of the solutions of (2.4). The pseudo-moduli are lifted at one loop, by the potential (1.5). The resulting vacua, in all these examples, always have $\langle X_i \rangle = 0$ as the minimum of the one-loop potential. The $U(1)_R$ symmetry is thus not spontaneously (nor explicitly) broken in these examples.

In the remainder of this section, we will discuss some particular examples in detail. Some of these examples have non-R global symmetries, which can be spontaneously broken. The results will be useful in the later sections, where we discuss modifications, with broken $U(1)_R$ symmetry.

2.1 The basic O’Raifeartaigh model OR_1

Let us consider the simplest case, $r = 2$, $s = 1$ in (2.1). Taking $g_1(\phi) = \frac{1}{2}h\phi^2 + f$ and $g_2(\phi) = m\phi$ gives, with a change of notation $\phi_1 = \phi$ and $\phi_2 = X_2$,

$$W_{(O'R)_1} = \frac{1}{2}hX\phi_1^2 + m\phi_1\phi_2 + fX. \tag{2.5}$$

In addition to the $U(1)_R$ symmetry, there is a \mathbb{Z}_2 symmetry, under which ϕ_1 and ϕ_2 are odd. Note that this model (2.5) has two dimensional parameters, m and f . They can be made naturally small if they are generated by dimensional transmutation of some added dynamics, e.g. as in [8].

The tree-level scalar potential is given by

$$V_{\text{tree}} = \left| \frac{1}{2} h \phi_1^2 + f \right|^2 + \left| h X \phi_1 + m \phi_2 \right|^2 + \left| m \phi_1 \right|^2. \quad (2.6)$$

There is an $r - s = 1$ complex dimensional pseudo-moduli space of vacua, with X and ϕ_2 constrained by the single condition (2.3), which here gives $hX\phi_1 + m\phi_2 = 0$. The ϕ_1 equation of motion (2.4) leads to two phases, depending on the value of

$$y \equiv \left| \frac{hf}{m^2} \right| \quad (2.7)$$

with a second order phase transition at $y = 1$. Let us now describe these phases in turn:

1. In the $y < 1$ phase, the potential is minimized along a pseudo-moduli space of supersymmetry breaking vacua, given by

$$\phi_1 = \phi_2 = 0, \quad X \text{ arbitrary.} \quad (2.8)$$

The \mathbb{Z}_2 symmetry is unbroken in this phase. The classical pseudo-moduli space degeneracy is lifted at one-loop by the Coleman-Weinberg potential (1.5). Near $X = 0$, the potential is

$$V_{\text{eff}}^{(1)}(X) = V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4), \quad (2.9)$$

where V_0 is a constant and

$$m_X^2 = \frac{1}{32\pi^2} |h^2 m^2| f_1(y) \quad (2.10)$$

$$f_1(y) \equiv y^{-1} \left((1+y)^2 \log(1+y) - (1-y)^2 \log(1-y) - 2y \right)$$

Higher loop corrections are suppressed by powers of h^2 . Since $f_1(y)$ is positive for all $y < 1$, the potential (2.9) has an $U(1)_R$ -preserving minimum at $X = 0$.

2. In the $y > 1$ phase, there are two disjoint pseudo-moduli spaces given by

$$\phi_2 = -\frac{hX}{m} \phi_1, \quad \phi_1 = \pm i \sqrt{\frac{2f}{h}} (1 - y^{-1}), \quad X \text{ arbitrary.} \quad (2.11)$$

The \mathbb{Z}_2 symmetry is spontaneously broken in this phase. The Coleman-Weinberg potential again takes the form (2.9), now with

$$m_X^2 = \frac{1}{8\pi^2} |h^2 m^2| f_2(y) \quad (2.12)$$

$$f_2(y) \equiv y^2 \log y - (y-1)^2 \log(y-1) - (y-1/2) (2 \log(y-1/2) + 1)$$

Again, since $f_2(y) > 0$ for all $y > 1$, the one-loop effective potential is minimized at $X = 0$, and the $U(1)_R$ remains unbroken.

2.2 A “cubic only” model, OR_2

Consider again (2.1) for $r = 2, s = 1$, now with $g_1(\phi) = \frac{1}{2}h_1\phi^2 + f$ and $g_2 = \frac{1}{2}h_2\phi^2$. For convenience, we will write it again as

$$W_{OR_2} = \frac{1}{2}h_1X\phi_1^2 + \frac{1}{2}h_2\phi_2\phi_1^2 + fX. \tag{2.13}$$

While (1.1) had two dimensionful parameters, the model (2.13) only has one, f . The tree-level scalar potential is

$$V_{\text{tree}} = \left| \frac{1}{2}h_1\phi_1^2 + f \right|^2 + \left| h_1X\phi_1 + h_2\phi_2\phi_1 \right|^2 + \left| \frac{1}{2}h_2\phi_1^2 \right|^2. \tag{2.14}$$

The potential (2.14) has a one-dimensional pseudo-moduli space of vacua, with supersymmetry broken by the non-zero vacuum energy $V_{\text{min}} \neq 0$. But the spectrum of massive fields is supersymmetric, and thus the loop corrections to the potential, such as (1.5), vanish. The reason is that there is a unitary change of variables, which preserves the Kähler potential, of the form,

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = U \begin{pmatrix} X \\ \phi_2 \end{pmatrix}, \tag{2.15}$$

with $U \in U(2)$, which can be used to take (2.13) into

$$W_{OR_2} \rightarrow \frac{1}{2}hY_1(\phi_1^2 + f_1) + f_2Y_2. \tag{2.16}$$

The theory thus decouples into two independent sectors. The first has two supersymmetric vacua, at $Y_1 = 0, \phi_1 = \pm i\sqrt{f_1}$. Supersymmetry is broken because of the decoupled Y_2 field, but that sector is a free field theory. So the model OR_2 is rather trivial.

2.3 Generalizations of OR_1 , with fields ϕ_1 and ϕ_2 in representations of a group

A generalization of (1.1), which we will use later, is to replace the fields ϕ_1 and ϕ_2 with representations \mathbf{r}_1 and \mathbf{r}_2 of some global symmetry group G . We keep X as a single field, in the singlet representation of G . We take the superpotential to still be given by (1.1), with the parameters h, m , and f in the singlet representation G . We choose the representations \mathbf{r}_1 and \mathbf{r}_2 such that

$$\mathbf{r}_1 \otimes \mathbf{r}_1 \supseteq \mathbf{1}, \quad \mathbf{r}_1 \otimes \mathbf{r}_2 \supseteq \mathbf{1}, \tag{2.17}$$

so that the superpotential (1.1) is G -invariant. (Note that \mathbf{r}_1 and \mathbf{r}_2 can in general be reducible representations.) In the notation of (2.1), we have $r = 1 + |\mathbf{r}_2|$ and $s = |\mathbf{r}_1|$.

For simplicity, let us consider the case $\mathbf{r}_1 = \mathbf{r}_2 \equiv \mathbf{r}$, which is taken to be an $|\mathbf{r}|$ dimensional (real) representation of G . The analysis is nearly identical to that of section 2.1. There are again two phases, depending on the value of the parameter y defined in (2.7).

1. When $y < 1$, the the absolute minimum of the tree-level potential occurs at

$$\phi_1 = \phi_2 = 0, \quad X \text{ arbitrary} \tag{2.18}$$

with $V_0 = |F_X|^2 = |f|^2$. The global symmetry G is thus unbroken in this phase. The tree-level mass matrices factorize into $|\mathbf{r}|$ copies of the basic O’Raifeartaigh

model (1.1), so the one-loop potential is just $|r|$ times that of the basic model (1.1), $V_{\text{eff}}^{(1)}(X; r) = |r|V_{\text{eff}}^{(1)}(X)$. In particular, near the origin, it is of the form (2.9) with

$$m_X^2 = \frac{|\mathbf{r}|h^2m^2}{32\pi^2}f_1(y) \tag{2.19}$$

2. When $y > 1$ the analog of the vacua (2.11) is

$$\phi_2 = -\frac{hX}{m}\phi_1, \quad (\phi_1^2)_1 = -\frac{2f}{h}(1 - y^{-1}), \quad \phi_1^\dagger = -\sqrt{\frac{f^*h}{fh^*}}\phi_1, \quad X \text{ arbitrary}, \tag{2.20}$$

where $(\phi_1^2)_1$ means the singlet component in (2.17). The value of the potential is $V_0 = |F_X|^2 + |F_{\phi_2}|^2$, with $|F_X|^2 = y^{-2}|f|^2$ and $|F_{\phi_2}|^2 = 2y^{-2}(y - 1)|f|^2$. Note that unlike in the $y < 1$ phase, for fixed X the solution to (2.20) is not unique or even discrete — that is to say, there can be an additional (compact) component to the pseudo-moduli space of vacua parameterized by ϕ_1 satisfying the second and third equations in (2.20). For simplicity, and since this is all we will need for the rest of the paper, we will limit our discussion to the special class of models where the solution to the ϕ_1 equations in (2.20) is unique up to global symmetries. In that case, the global symmetry G is spontaneously broken in this phase, by $\langle \phi_1 \rangle \neq 0$, to a subgroup $H \subset G$. The scale of the $G \rightarrow H$ symmetry breaking varies along the pseudo-moduli space, increasing with $|X|$, because of the ϕ_2 expectation value in (2.20). Note that there is no value of X for which G is unbroken because ϕ_1 never vanishes.

The pseudo-moduli space is lifted at one loop, but there is, in the full quantum theory, a compact moduli space of vacua, the Goldstone boson manifold G/H , of real dimension $|G/H|$. Decomposing \mathbf{r} into H representations, it contains $|\mathbf{r}| - |G/H|$ singlets. Thus, the classical mass spectrum of the ϕ_1 and ϕ_2 fields coincides with that of $|\mathbf{r}| - |G/H|$ decoupled copies of the basic O’Raifeartaigh model with $y > 1$, and $|G/H|$ copies of the basic O’Raifeartaigh model with $y = 1$ (these supply the needed massless Goldstone bosons). So the one-loop potential (1.5) is

$$V_{\text{eff}}^{(1)}(X; \mathbf{r}, y > 1) = (|\mathbf{r}| - |G/H|)V_{\text{eff}}^{(1)}(X; y > 1) + |G/H|V_{\text{eff}}^{(1)}(X; y = 1). \tag{2.21}$$

In particular, the minimum is at $X = 0$, around which the potential takes the form (2.9) with

$$m_X^2 = \frac{h^2m^2}{8\pi^2} \left((|\mathbf{r}| - |G/H|)f_2(y) + \frac{1}{2}|G/H|(\log 4 - 1) \right). \tag{2.22}$$

We stress that this analysis only applies to the special class of models where ϕ_1 is completely specified up to global symmetries by the equations (2.20).

In section 4, we will reconsider these models, with the symmetry G (or a subgroup) replaced with a gauge, rather than global, symmetry. In the $y < 1$ phase, G remains unbroken, so the 1-loop potential is unaffected by the gauging and the $U(1)_R$ symmetry

remains unbroken. (It is amusing to note that, even if $U(1)_R$ were to be spontaneously broken, the messenger mass matrix \mathcal{M} in all of these models satisfies $\det \mathcal{M} = \text{const}$ [17], and so the gauginos would remain massless to leading order in the SUSY breaking.) On the other hand, in the $y > 1$ phase G is broken, so here the 1-loop potential is affected by the gauging and can have a minimum away from the origin.

2.4 Generalizations of OR_2 , with fields ϕ_1 and ϕ_2 representations of a group

We noted in section 2.2 that the model with superpotential (2.13) was rather trivial, because it decoupled into a supersymmetric sector and a free field theory. We now consider generalizing the model, with the same superpotential (2.13), by making ϕ_1 and ϕ_2 representations \mathbf{r}_1 and \mathbf{r}_2 of a global symmetry group G . Again, we keep X as a singlet representation of G . The couplings h_1 , h_2 , and f are singlets of G , and the superpotential (2.13) is G invariant if the representations satisfy

$$\mathbf{r}_1 \otimes \mathbf{r}_1 \supseteq \mathbf{1} \oplus \overline{\mathbf{r}}_2. \tag{2.23}$$

When $\mathbf{r}_2 \neq \mathbf{1}$, there is no longer a unitary change of variables generalizing (2.15), to bring the theory to a decoupled form analogous to (2.16). So this modified theory is nontrivial.

These models are a particular example of the general class of models (2.1), with $r = 1 + |\mathbf{r}_2|$ and $s = |\mathbf{r}_1|$. If the superpotentials were generic, and unrestricted by the G symmetry, we would then have that supersymmetry is broken if $r > s$, i.e. if $|\mathbf{r}_2| \geq |\mathbf{r}_1|$, and unbroken otherwise. Let us consider this in more detail, accounting for the particular, G symmetric, form of the superpotential. The tree-level potential is

$$V_{\text{tree}} = \left| \frac{1}{2} h_1 (\phi_1^2)_{\mathbf{1}} + f \right|^2 + \left| h_1 X \phi_1 + h_2 (\phi_2 \phi_1)_{\mathbf{r}_1} \right|^2 + \left| \frac{1}{2} h_2 (\phi_1^2)_{\overline{\mathbf{r}}_2} \right|^2, \tag{2.24}$$

where $(\phi_1^2)_{\mathbf{1}}$ and $(\phi_1^2)_{\overline{\mathbf{r}}_2}$ are the singlet and the $\overline{\mathbf{r}}_2$ representations in (2.23), respectively. We can always choose the X and ϕ_2 expectation values such that the middle term in (2.24) vanishes,

$$h_1 X \phi_1 + h_2 (\phi_2 \phi_1)_{\mathbf{r}_1} = 0. \tag{2.25}$$

This is $|\mathbf{r}_1|$ conditions on the $1 + |\mathbf{r}_2|$ fields X and ϕ_2 , so (2.25) is satisfied on a (pseudo)moduli space of superficial dimension $|\mathbf{r}_2| + 1 - |\mathbf{r}_1|$ (the actual dimension can differ from that). These models will break supersymmetry iff there is no simultaneous solution to

$$(\phi_1^2)_{\mathbf{1}} \neq 0, \quad (\phi_1^2)_{\overline{\mathbf{r}}_2} = 0. \tag{2.26}$$

Whether or not there are supersymmetric vacua, where (2.26) can be satisfied, depends on the representations \mathbf{r}_1 and \mathbf{r}_2 .

The X and ϕ_2 equations of motion are satisfied on the space (2.25). The ϕ_1 equations of motion are cubic, and one solution is always $\langle \phi_1 \rangle = 0$. However, this is always a saddle point of the potential, as seen by expanding (2.24) to quadratic order around $\phi_1 = 0$, and noting that there is a tachyonic mode, where the first term in (2.24) can be reduced without affecting the other terms. The minima of the potential are given by the non-zero solutions, $\phi_1 = \phi_1^0 \neq 0$. Expanding around such a solution, the symmetry G is broken to some

subgroup H . This subgroup will in general depend on ϕ_1^0 ; in particular, it will be enhanced at special values of ϕ_1^0 . It is generally the case that the points of enhanced unbroken symmetry are extrema of the effective potential, and in many (if not all) examples, the point of maximal unbroken global symmetry is a local minimum.

Let us consider some examples. Take $G = \text{SU}(N)$ and let \mathbf{r}_1 and \mathbf{r}_2 be the adjoint representation. This is a model considered in [18] (with $G = \text{SU}(5)$ gauged, and identified with the GUT gauge group). In this case, there is no solution of (2.26), so supersymmetry is broken. Another example is $G = \text{SU}(N_f) \times \text{SU}(N)$, with $\mathbf{r}_1 = (\mathbf{N}_f, \mathbf{N}) \oplus (\overline{\mathbf{N}}_f, \overline{\mathbf{N}})$ and $\mathbf{r}_2 = (\mathbf{N}_f^2 - \mathbf{1}, \mathbf{1})$. This example is the supersymmetry breaking model analyzed in [7], with $\phi_1 = \varphi \oplus \tilde{\varphi}$, $\phi_2 = \Phi - \frac{1}{N_f} \text{Tr } \Phi$, and $X = \text{Tr } \Phi$. This model breaks supersymmetry if $N_f > N$, and in this model, the general condition (2.26) is the “rank condition” of [7].

To summarize, in this class of “cubic only” models, there is a pseudo-moduli space given by the solutions to (2.25) and to the ϕ_1 equations of motion. Along this pseudo-moduli space, the global symmetry G is always spontaneously broken by $\phi_1 = \phi_1^0 \neq 0$ to some subgroup H , for all values of the parameters in the superpotential. Therefore, regardless of where the effective potential is minimized on this pseudo-moduli space, there is always a compact moduli space of vacua, a Goldstone boson manifold G/H . This should be contrasted with the OR_1 class of models discussed in the previous subsection, where the symmetry breaking phase depended on the parameter $y = hf/m^2$.

3. Metastable SUSY breaking in a modified O’Raifeartaigh model

In section 2, we saw that for all values of the couplings, the vacuum of the basic O’Raifeartaigh model occurred at $\langle X \rangle = 0$, where $U(1)_R$ is unbroken. Let us now consider what happens when a small, explicit R-symmetry breaking operator is added to the superpotential (1.1). Generically, the presence of such an operator introduces SUSY vacua elsewhere in field space, rendering the SUSY-breaking vacuum metastable. However, as long as the coupling is small, these vacua will be well separated and the SUSY-breaking vacuum parametrically long-lived.

To be concrete, let us consider the superpotential

$$W = \frac{1}{2}hX\phi_1^2 + m\phi_1\phi_2 + fX + \frac{1}{2}\epsilon m\phi_2^2 \tag{3.1}$$

with $|\epsilon| \ll 1$. The classical scalar potential is now

$$V_{\text{tree}} = \left| \frac{1}{2}h\phi_1^2 + f \right|^2 + \left| hX\phi_1 + m\phi_2 \right|^2 + \left| m\phi_1 + \epsilon m\phi_2 \right|^2. \tag{3.2}$$

There are two supersymmetric vacua, at

$$\langle \phi_1 \rangle_{\text{susy}} = \pm \sqrt{-2f/h}, \quad \langle \phi_2 \rangle_{\text{susy}} = \mp \frac{1}{\epsilon} \sqrt{-2f/h}, \quad \langle X \rangle_{\text{susy}} = \frac{m}{h\epsilon}. \tag{3.3}$$

For small ϵ , the supersymmetric vacua (3.3) have X far from the origin. As $\epsilon \rightarrow 0$, these supersymmetric vacua are pushed to infinity.

In addition to these supersymmetric vacua, the scalar potential (3.2) is approximately minimized along the pseudo-moduli spaces of the two phases. For $y < 1$, the pseudo-moduli space of the $\epsilon = 0$ theory (2.8) remains an extremum of the potential of the $\epsilon \neq 0$ theory. The classical masses of the fermionic and scalar components of ϕ_1 and ϕ_2 around (2.8) are given by

$$m_0^2 = \frac{1}{2} \left(|hX|^2 + |m|^2(2 + |\epsilon|^2) + \eta|hf| \right. \\ \left. \pm \sqrt{(|hX|^2 + |m|^2(2 + |\epsilon|^2) + \eta|hf|)^2 - 4m^2(|hX\epsilon - m|^2 + \eta|hf|(1 + |\epsilon|^2))} \right) \\ m_{1/2}^2 = m_0^2|_{f=0} \tag{3.4}$$

where $\eta = \pm 1$. In order for the pseudo-moduli space of supersymmetry breaking vacua to be locally stable, without tachyonic modes, the eigenvalues (3.4) must all be positive. There are indeed no tachyonic modes for a range of the pseudo-modulus X :

$$\left| 1 - \frac{\epsilon h X}{m} \right|^2 > (1 + |\epsilon|^2) y \tag{3.5}$$

Outside of the range (3.5), there is one tachyonic mode in (3.4), and the pseudo-moduli space is there locally unstable, as there the fields can roll down the tachyonic direction to the supersymmetric vacua (3.3). In the region where (3.5) is satisfied, the pseudomoduli space is locally stable. Note that this region includes a large neighborhood of the origin $X = 0$ for all

$$y < 1/(1 + |\epsilon|^2). \tag{3.6}$$

The classical pseudo-moduli space degeneracy is lifted by the 1-loop effective potential, which we compute using (1.5), with the masses (3.4). We restrict our attention to the range (3.5) and (3.6), where the pseudo-moduli space is locally stable. The effective potential $V_{\text{eff}}(X)$ thus computed is found to have a minimum near the origin. In particular, expanding near the origin, the result is

$$V_{CW}^{(1)} = V_0 + m_X^2 |X - X_{\text{min}}|^2 + \mathcal{O}(\epsilon^2, |X - X_{\text{min}}|^4) \tag{3.7}$$

To order ϵ , the local minimum is moved from the origin to

$$X_{\text{min}} = -\frac{\epsilon m}{h} \frac{(1+y)\log(1+y) - (1-y)\log(1-y) - 2y}{(1+y)^2\log(1+y) - (1-y)^2\log(1-y) - 2y} + \mathcal{O}(\epsilon^3). \tag{3.8}$$

This is a metastable vacuum, with supersymmetry broken. The light spectrum in this vacuum consists of the massless Goldstone fermion, ψ_X . All other modes have $\mathcal{O}(\epsilon^0)$ masses. In particular, the pseudo-modulus X has mass m_X as in (2.11), and there is no light (pseudo) Goldstone boson ‘‘R-axion.’’

For $y > 1$, the pseudo-moduli space of the $\epsilon = 0$ theory (2.11) is no longer an exact extremum of the potential; instead, there is an $\mathcal{O}(\epsilon)$ tadpole for X . However, we expect that for $\epsilon \ll h$, this tadpole will be stabilized by a positive m_X^2 at one-loop, approximately (2.13) to leading order in ϵ . To make this more precise, let us integrate out ϕ_1 and ϕ_2 exactly at

tree-level. Their mass-squareds are complicated functions of h , X , m , f and ϵ , analogous to (3.4). Curiously, they are positive as long as the *reverse* of (3.5) is satisfied:

$$\left|1 - \frac{\epsilon h X}{m}\right|^2 < (1 + |\epsilon|^2) y \tag{3.9}$$

For $y > 1$, this inequality always includes a neighborhood around $X = 0$. Integrating out ϕ_i , we obtain an effective potential for X , which is an expansion in powers of ϵX , ϵX^* :

$$V_{\text{eff}} = -\frac{m^3 \epsilon (y - 1)}{32\pi^2 h} (X + X^*) + \mathcal{O}(\epsilon^2 X^2, \epsilon^2 (X^*)^2, \epsilon^2 |X|^2) \tag{3.10}$$

Adding to this the one-loop potential (2.9)(2.13), we see that there is indeed a local minimum at

$$X_{\text{min}} = \frac{\epsilon m (y - 1)}{h^3 f_2(y)} (1 + \mathcal{O}(h^2)) \tag{3.11}$$

Here the $\mathcal{O}(h^2)$ corrections include not only the corrections to m_X^2 , but also the $\mathcal{O}(\epsilon h)$ tadpole term that is expected to appear in the one-loop Coleman-Weinberg potential, due to the explicitly broken R-symmetry.

Finally, let us consider the lifetime of these metastable SUSY-breaking vacua. For $\epsilon \ll 1$, the metastable vacuum (3.8)(3.11) is widely separated from the supersymmetric vacua (3.3) by an $\mathcal{O}(1/\epsilon)$ distance in field space. This ensures that the supersymmetry breaking vacuum is parametrically long lived as $\epsilon \rightarrow 0$. In particular, the bounce action scales as $S_{\text{bounce}} \sim \epsilon^{-\alpha}$ for some $\alpha > 0$, and can be made arbitrary large for ϵ sufficiently small.

4. Spontaneous $U(1)_R$ breaking in a gauged $SO(N)$ model

Having analyzed a toy model with explicit $U(1)_R$ breaking, we now turn to a model with spontaneous $U(1)_R$ breaking. As described in the introduction, this can be achieved by gauging a global symmetry G in an O’Raifeartaigh type model. If G is spontaneously broken along the pseudo-moduli space, then for appropriate values of the parameters, the pseudo-modulus can get a negative mass-squared around the origin.

Specifically, the models we will consider in this section are those of section 2.3, with $G = SO(N)$ and the fields $\phi_{1,2}$ transforming in the fundamental representation.⁴ Thus, our superpotential is given by

$$W = \frac{1}{2} h X \vec{\phi}_1^2 + m \vec{\phi}_1 \cdot \vec{\phi}_2 + f X \tag{4.1}$$

For $N = 2$ this model was studied recently in [9]. For simplicity, we will take all the couplings to be real and positive throughout this section, which can be always be done via field phase rotations. In section 2.3, we saw that without gauging the global symmetry, the model always has a $U(1)_R$ preserving vacuum at $X = 0$. Now let us analyze what happens

⁴For $N = 6$, in the $y \rightarrow \infty$ limit, this model reduces to the IR description of the ITTY model [25, 26].

with we gauge the full $SO(N)$ global symmetry. This introduces D-terms into the scalar potential, $V_D = \frac{1}{2}g^2(D^{(ab)})^2$ where

$$D^{(ab)} = \phi_1^\dagger T^{(ab)} \phi_1 + \phi_2^\dagger T^{(ab)} \phi_2 \tag{4.2}$$

and $T^{(ab)}$ are the generators of $SO(N)$,

$$T^{(ab)}_{cd} = \delta_c^a \delta_d^b - \delta_d^a \delta_c^b \tag{4.3}$$

with $1 \leq a < b \leq N$. Substituting (2.18) and (2.20) into (4.2), we see that the D-terms vanish identically along the pseudo-moduli spaces of both the $y < 1$ and the $y > 1$ phases. Therefore, (2.18), (2.20) remain the absolute minima of the tree-level scalar potential in their respective phases, even after gauging the $SO(N)$.

In the $y < 1$ phase, the $SO(N)$ symmetry is unbroken, so the one-loop effective potential is independent of g as discussed in the introduction. Thus the one-loop effective potential is still given by (2.19), with an R-preserving minimum at $X = 0$.

In the $y > 1$ phase, however, the $SO(N)$ symmetry is spontaneously broken, so now the potential depends on g . At small $|X|$, we have instead of (2.22)

$$V_{CW}^{(1)} = const. + \frac{h^2 m^2}{16\pi^2} \left((N-1)f_3(\eta) + 2f_2(y) \right) |X|^2 + \mathcal{O}(|X|^4) \tag{4.4}$$

where

$$f_3(\eta) = \frac{1}{1+2\eta^2} \left(6\eta^4(1+2\eta^2) \log 2\eta^2 + 2(1+\eta^4+2\eta^6) \log(2+2\eta^2) - 2(1+2\eta^2)(1+4\eta^4) \log(1+2\eta^2) - (1-2\eta^2)^2 \right) \tag{4.5}$$

and we have defined

$$\eta = \frac{g}{h} \sqrt{2(y-1)} \tag{4.6}$$

Now since $f_2(y)$ is bounded and positive for $y > 1$, while $f_3(\eta)$ is positive at $\eta = 0$ but is unbounded from below as $\eta \rightarrow \infty$, we see that there must exist some $\eta_{\min}(y)$ such that when

$$\eta > \eta_{\min}(y) \tag{4.7}$$

the pseudo-modulus is tachyonic at $|X| = 0$.

Although we have shown that for $\eta > \eta_{\min}$, the R-preserving vacuum at the origin is destabilized, it remains to be seen whether there is a vacuum for any $|X| \neq 0$, or whether the potential simply runs away to infinity. At large X , the potential (1.5) has the behavior

$$V_{CW}^{(1)} \propto h^2 \left(N - 4(N-1)\eta^2 \right) \log \frac{|X|}{M_{\text{cutoff}}} + \mathcal{O}(|X|^{-1}). \tag{4.8}$$

This also follows from (1.3). The tree-level vacuum energy density is $V_0 = |F_X|^2 + |F_{\phi_2}|^2$, with $|F_X|^2 = y^{-2}|f|^2$ and $|F_{\phi_2}|^2 = 2y^{-2}(y-1)|f|^2$. The renormalization of these terms leads to (1.3) with the anomalous dimension of the pseudo-modulus given by

$$\gamma = \frac{|F_X|^2 \gamma_X + |F_{\phi_2}|^2 \gamma_{\phi_2}}{|F_X|^2 + |F_{\phi_2}|^2} = \frac{\gamma_X + 2(y-1)\gamma_{\phi_2}}{2y-1}, \tag{4.9}$$

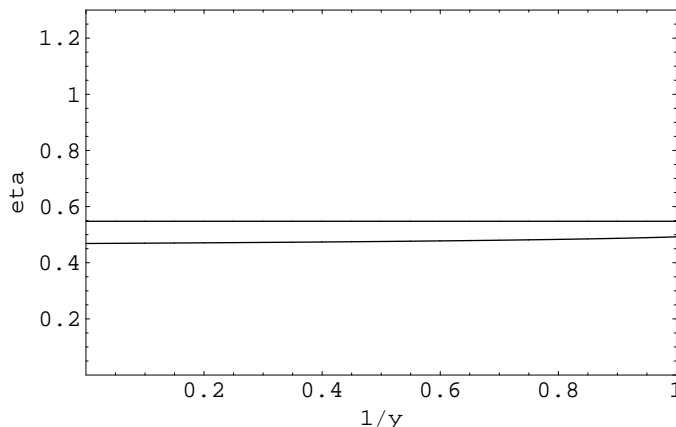


Figure 1: A plot of $\eta_{\min}(y)$ (bottom curve) and η_{\max} (top curve) vs. y^{-1} in the gauged $\text{SO}(N = 6)$ model. The window (4.12) is for η in the region between the two solid lines.

Here γ_X and γ_{ϕ_2} are the anomalous dimensions of those fields, given at one loop by

$$\gamma_X^{(1)} = \frac{h^2 N}{32\pi^2}, \quad \gamma_{\phi_2}^{(1)} = -\frac{g^2(N-1)}{8\pi^2}, \quad (4.10)$$

where the factor of $N - 1$ comes from the Casimir $C_2(\mathbf{r}) = |G|T(\mathbf{r})/|\mathbf{r}|$. Using (4.10) in (4.9), we thus obtain for the anomalous dimension of the pseudo-modulus $\gamma = h^2(N - 4\eta^2(N - 1))/32\pi^2(2y - 1)$. Using this in (1.3) agrees with (4.8).

So we see from (4.8) that as long as

$$\eta < \eta_{\max} = \frac{1}{2}\sqrt{\frac{N}{N-1}} \quad (4.11)$$

the potential curves up at infinity, so $\langle X \rangle$ cannot be too large. Our interest is in the window where both (4.7) and (4.11) are satisfied,

$$\eta_{\min}(y) < \eta < \eta_{\max}. \quad (4.12)$$

In this case, there is a SUSY-breaking, R-breaking vacuum at some finite $\langle X \rangle$ which is not zero, and also not hierarchically large. A plot of $\eta_{\min}(y)$ and η_{\max} vs. y^{-1} is shown in figure 1, for $N = 6$. This figure also illustrates a general feature of $\eta_{\min}(y)$ — it is a monotonically increasing function of y^{-1} .

Let us make a few comments on the window (4.12):

1. The window is non-empty, for all N and $y > 1$. One can verify this by, for instance, checking that $|X|$ is always tachyonic at the origin when $\eta = \eta_{\max}$.
2. The window is generally quite small. Figure 1 shows the typical values of $\eta_{\min} \approx 0.47$ (the dependence on N is minimal), while according to (4.11), $\eta_{\max} \approx 0.5 - 0.6$ for largish N . So the window in η is typically a size of order $\Delta\eta \approx 0.05 - 0.1$. In addition, for η close to η_{\max} , the existence of the minimum of the potential depends sensitively on the large X behavior of the potential (4.8). This is suppressed not only by the

loop-counting parameter h^2 , but also by $\eta_{\max} - \eta < \Delta\eta \ll 1$. Since the two-loop correction to the potential will not be suppressed by this additional factor of $\Delta\eta$, in order for us to be able to trust the one-loop approximation, we need h (and hence g) to be smaller than the naive perturbation expansion would suggest.

3. The existence of this window of spontaneous $U(1)_R$ breaking depends on the details of the full Coleman-Weinberg potential, and not just its leading logarithm (4.8). Correspondingly, the scale of $U(1)_R$ breaking $\langle X \rangle$ is not hierarchically large, but rather is $\mathcal{O}(M)$ where M is some characteristic combination of the mass scales in the superpotential.
4. Because the R-symmetry is spontaneously broken, the massless spectrum includes a real scalar, the R-axion, in addition to the Goldstino and the $SO(N-1)$ gauge fields. The $SO(N-1)$ gauginos are massless at tree-level, but they will pick up a Majorana mass term at one-loop. (One possibility that we have not checked in this example is that the gaugino masses vanish to leading order in the SUSY breaking. This happens in O’Raifeartaigh models without spontaneous gauge symmetry breaking, as we discuss at the end of section 2.3.)

Finally, let us discuss what happens for $\eta > \eta_{\max}$. This regime corresponds to the “inverted hierarchy” phase first studied by Witten in [18]. Here the 1-loop potential has runaway at infinity, and there may or may not be a vacuum at exponentially large fields, depending on the details of the renormalization group equations for g and h . While the “inverted hierarchy” and its uses for model building are well-known (see e.g. [18–21]), the uses of the non-hierarchical phase have been relatively unexplored (see however [22], and the more recent work of [9, 14]). However, if the non-hierarchical phase always occurs in a small window of couplings such as (4.12), then the usefulness of this phase for (natural) model building might be limited.

4.1 Gauged $SO(n) \subset SO(N)$ — tree-level vacuum alignment/mis-alignment

In this subsection, we will analyze the $SO(N)$ model with a subgroup $SO(n) \subset SO(N)$ gauged. As we shall see, the model becomes much more complicated, so we will focus mainly on a few qualitative physics points and be brief with the technical details.

Since we have broken the $SO(N)$ global symmetry explicitly by gauging an $SO(n)$ subgroup, with $SO(n)$ gauge coupling $g \neq 0$, we should consider the most general $SO(n) \times SO(N-n)$ invariant superpotential of the form (1.1). This is given by:

$$W = \frac{1}{2}hX\phi_1^2 + \frac{1}{2}\tilde{h}X\tilde{\phi}_1^2 + m\phi_2 \cdot \phi_1 + \tilde{m}\tilde{\phi}_2 \cdot \tilde{\phi}_1 + fX \tag{4.13}$$

where ϕ_1, ϕ_2 ($\tilde{\phi}_1, \tilde{\phi}_2$) transform in the fundamental of $SO(n)$ ($SO(N-n)$). Because $SO(N)$ is not a symmetry, we generally have $h \neq \tilde{h}$ and $m \neq \tilde{m}$ (taking them to be equal would not be preserved by renormalization.). Because of the tree-level interactions, there is no limit where $SO(N)$ is restored, even as an accidental symmetry. So there is no issue of dynamical vacuum alignment here. Whether the vacuum aligns to break, or not break,

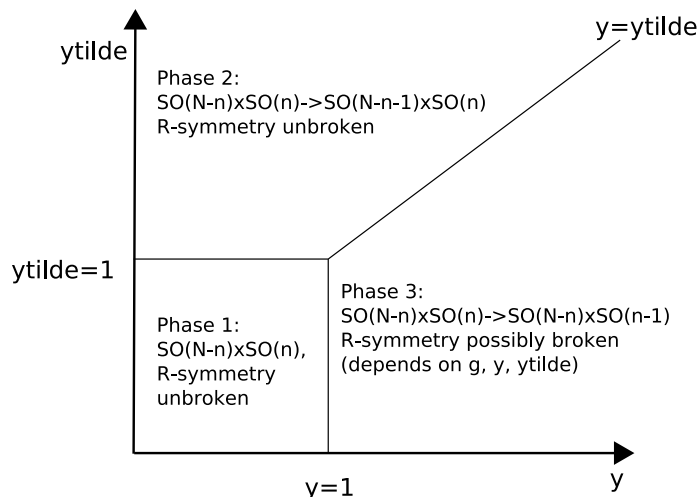


Figure 2: A phase diagram for the gauged $SO(n) \subset SO(N)$ O’Raifeartaigh model.

the $SO(n)$ gauge symmetry is determined entirely at tree-level, by the couplings in the superpotential (4.13). Indeed, there are now three different phases depending on the ratios $y = |hf/m^2|$ and $\tilde{y} = |\tilde{h}f/\tilde{m}^2|$ (these are summarized in figure 2):

1. When $y, \tilde{y} < 1$, the F and D-terms are minimized with

$$\phi_i = \tilde{\phi}_i = 0, \quad X \text{ arbitrary} \quad (4.14)$$

with $V_0 = f^2$. Since the gauge symmetry is unbroken, the one-loop Coleman-Weinberg potential is independent of the gauge coupling, and so it reduces to an obvious generalization of (2.19):

$$m_X^2 = \frac{1}{32\pi^2} \left(nh^2m^2f_1(y) + (N-n)\tilde{h}^2\tilde{m}^2f_1(\tilde{y}) \right) \quad (4.15)$$

So in this phase the R-symmetry remains unbroken.

2. $\tilde{y} > 1, y < \tilde{y}$. Now the scalar potential is minimized along

$$\phi_i = 0, \quad \tilde{\phi}_2 = -\frac{\tilde{h}X}{\tilde{m}}\tilde{\phi}_1, \quad \tilde{\phi}_1^2 = -|\tilde{\phi}_1|^2 = -\frac{2f}{\tilde{h}}(1-\tilde{y}^{-1}), \quad X \text{ arbitrary} \quad (4.16)$$

Since $\phi_i = 0$, the pseudo-moduli space preserves the full $SO(n)$ gauge symmetry. So this phase corresponds to tree-level vacuum alignment. Since the gauge symmetry is again unbroken, the one-loop CW potential is independent of g . In fact, it is a straightforward generalization of (2.22),

$$m_X^2 = \frac{1}{32\pi^2} \left(h^2m^2nf_1(y/\tilde{y}) + \tilde{h}^2\tilde{m}^2 \left[2(N-n-1)(\log 4 - 1) + 4f_2(\tilde{y}) \right] \right) \quad (4.17)$$

and consequently, the R-symmetry remains unbroken.

3. $y > 1, \tilde{y} < y$:

$$\tilde{\phi}_i = 0, \quad \phi_2 = -\frac{hX}{m}\phi_1, \quad \phi_1^2 = -|\phi_1^2| = -\frac{2f}{h}(1-y^{-1}), \quad X \text{ arbitrary} \quad (4.18)$$

Since $\phi_1 \neq 0$, the pseudo-moduli space spontaneously breaks the gauge symmetry from $SO(n) \rightarrow SO(n-1)$. So this phase corresponds to tree-level vacuum misalignment. Now the one-loop potential depends on g , and its form is a generalization of (4.4):

$$m_X^2 = \frac{1}{32\pi^2} \left(h^2 m^2 \left[2(n-1)f_3(\eta) + 4f_2(y) \right] + \tilde{h}^2 \tilde{m}^2 (N-n)f_1(\tilde{y}/y) \right), \quad (4.19)$$

The qualitative features of this potential are the same as for (4.4). In particular, there will be a (possibly small) range of parameters where the R-breaking vacuum exists at $X \sim \mathcal{O}(M)$, where M is again some characteristic combination of the mass scales. In addition, there will be an “inverted hierarchy” phase where X is exponentially larger than M .

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